Chapter 8
Rotational Motion I Chapter Review

EQUATIONS:

- $v = r\,\omega$ [This is the relationship between the magnitude of the velocity v of a point moving in a circular path of radius r and the magnitude of the motion's angular velocity ω about the center of motion. Note that the center must be a *fixed point* in the system.]

- $a = r\,\alpha$ [This is the relationship between the magnitude of the acceleration a of a point moving in a circular path of radius r and the magnitude of the motion's angular acceleration α about the center of motion. Note that the center must be a *fixed point* in the system.]

- $\omega = -5\boldsymbol{i}$ [This is an angular velocity vector, where 5 is the magnitude, $\boldsymbol{i}$ is the direction of the *axis about which the rotation occurs*, and the negative sign signifies that the rotation is *clockwise* as viewed from the $+\boldsymbol{i}$ side of the coordinate axis.]

- $\alpha_{instantaneous} = \alpha_{average} = \alpha$ [If the angular acceleration is CONSTANT, the instantaneous angular acceleration and the average angular acceleration are equal. In such cases, both angular acceleration terms are characterized with an α.]

- $\alpha = \frac{\Delta\omega}{\Delta t}$, or $\omega_2 = \omega_1 + \alpha t$ [This rotational kinematic equation comes from the fact that the instantaneous angular acceleration α and the average angular acceleration $\frac{\Delta\omega}{\Delta t}$ over any interval will be the same, given the *angular acceleration* is *constant*. REMEMBER whenever you see a *time variable*, whether presented as t or Δt, you are *always* dealing with a TIME INTERVAL.]

- $\Delta\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ or $\theta_2 = \theta_1 + \omega_1 t + \frac{1}{2}\alpha t^2$ [This rotational kinematic equation relates angular displacement $\Delta\theta$ (i.e., the change of angular position between *time 1* and *time 2*), the initial angular velocity ω_1 (i.e., the angular velocity at *time 1*), the angular acceleration α, and the time interval t.]

- $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha(\theta_2 - \theta_1)$ [This rotational kinematic equation relates angular velocities at two different points in time to the angular acceleration α and the angular displacement $\Delta\theta$ during that interval.]

- $\omega_{avg} = \frac{\omega_2 + \omega_1}{2}$ [This rotational kinematic equation is RARELY USED. It relates the average angular velocity ω_{avg} over an interval to the initial and final angular velocities--ω_1 and ω_2--associated with that interval.]

- $\Delta\theta = \omega_{avg}\Delta t$ [This rotational kinematic equation is RARELY USED. It relates the average angular velocity ω_{avg} over time interval Δt to the angular displacement $\Delta\theta$ during that time interval.]

- $I = \sum_{i=1}^{n} m_i r_i^2$ [This is the definition of the moment of inertia of n discrete pieces of mass about a given axis, where m_i is the i^{th} mass in the system and r_i is the perpendicular distance (i.e., the shortest distance) between that mass and the axis in question.]

- $I = mr^2$ [This is the moment of inertia of a point mass m about an axis, where r is the perpendicular distance from the mass to the axis in question.]

- $I = \int r^2 dm$ [This expression is used to derive the moment of inertia of a continuous, solid object about some axis, where r is the perpendicular distance from the mass dm to the axis in question.]

- $\lambda = \frac{mass}{unit\ length} = \frac{dm}{dl}$ [Called a *linear density function*, λ (lambda) is a RATIO that quantifies the amount of *mass per unit length* there is along some linear structure. Differentially, $\lambda = \frac{dm}{dl}$ with dm being the differential mass involved in a differential length dl. The expression is useful because it allows you to express dm in terms of λ and dl, or $dm = \lambda dl$. This function is used whenever there is mass variation in one dimension only, specifically for rod-like structures. If the structure is homogeneous (i.e., the mass is uniformly distributed throughout), λ also equals the total mass in the structure divided by the total length of the structure, or M/L. If the structure is inhomogeneous, M/L is nonsense and a function must be provided for λ (i.e., something like $\lambda = kx$, where k is a constant and x is a variable that defines the distance between dm and an axis of interest). In any case, the $dm = \lambda dl$ relationship is ALWAYS true.]

- $\sigma = \frac{mass}{unit\ area} = \frac{dm}{dA}$ [Called an *area density function*, σ (sigma) is a RATIO that quantifies the amount of *mass per unit area* there is behind a given area on the surface of a body. Differentially, $\sigma = \frac{dm}{dA}$ with dm being the differential mass behind the differential surface area dA. This function is used whenever there is mass variation in two dimensions, or in uniformly distributed three-dimensional situations that just *look* easier to do in two dimensions--a rectangular solid is a good example. The expression is useful because it allows you to write dm in terms σ and dA, or $dm = \sigma dA$. If the structure is homogeneous, σ is equal to the total mass within the structure divided by the total surface area of the structure, or M/A. If the structure is inhomogeneous, M/A is nonsense and a function is required to tell you how the density acts from point to point (something like $\sigma = kr$, where k is a constant and r is a distance variable that makes sense relative to the coordinate system). In any case, the $dm = \sigma dA$ expression is ALWAYS true.]

- $\rho = \frac{mass}{unit\ volume} = \frac{dm}{dV}$ [Called a *volume density function*, ρ (rho) is a RATIO that quantifies the amount of *mass per unit volume* associated with a distribution of mass. Differentially, $\rho = \frac{dm}{dV}$ with *dm* being the differential mass associated with the differential volume dV. The expression is useful because it allows you to write *dm* in terms of ρ and dV, or $dm = \rho dV$. If the structure is homogeneous (i.e., the mass is uniformly distributed throughout), ρ also equals the total mass in the structure divided by the total volume, or *M/V*. If, on the other hand, the structure is inhomogeneous, *M/V* is nonsense and a function for ρ must be provided (i.e., something like $\rho = kr$, where *k* is a constant and *r* is a variable that defines the distance between *dm* and an axis of interest). In any case, the $dm = \rho dV$ expression is ALWAYS true.]

COMMENTS, HINTS, and THINGS to be aware of:

- The **rotational kinematic equations**--what are they?
  --These are rotational relationships between the angular displacement $\Delta\theta$, angular velocity ω, angular acceleration α, and time *t*, that are applicable ONLY when the angular acceleration is CONSTANT (i.e., when α is not, say, a function of θ).

- $v = r\omega$ is predicated on the **assumption** that the angular velocity of an object is measured as the object sweeps circularly about a **fixed point**. The same is true of $a = r\alpha$.

- In theory, the **integral form** of the **moment of inertia** expression says the following: Take all of the mass you can find that is located within a tiny differential volume *dV* a distance *r* units from the axis in question (or, if you are using an area density function, within a tiny differential area *dA* a distance *r* units from the axis . . . or, if you are working with a linear density function, in a tiny differential length *dr* a distance *r* units from the axis), call that differential mass *dm*, multiply *dm* by r^2, do that for all possible *r's*, then sum (i.e., integrate--the final expression will look like $I = \int r^2 dm$).